

# MULTI AGENT COOPERATIVE PATH PLANNING VIA MODEL PREDICTIVE CONTROL

Christian Kallies, Institut für Flugführung, 23.05.2023



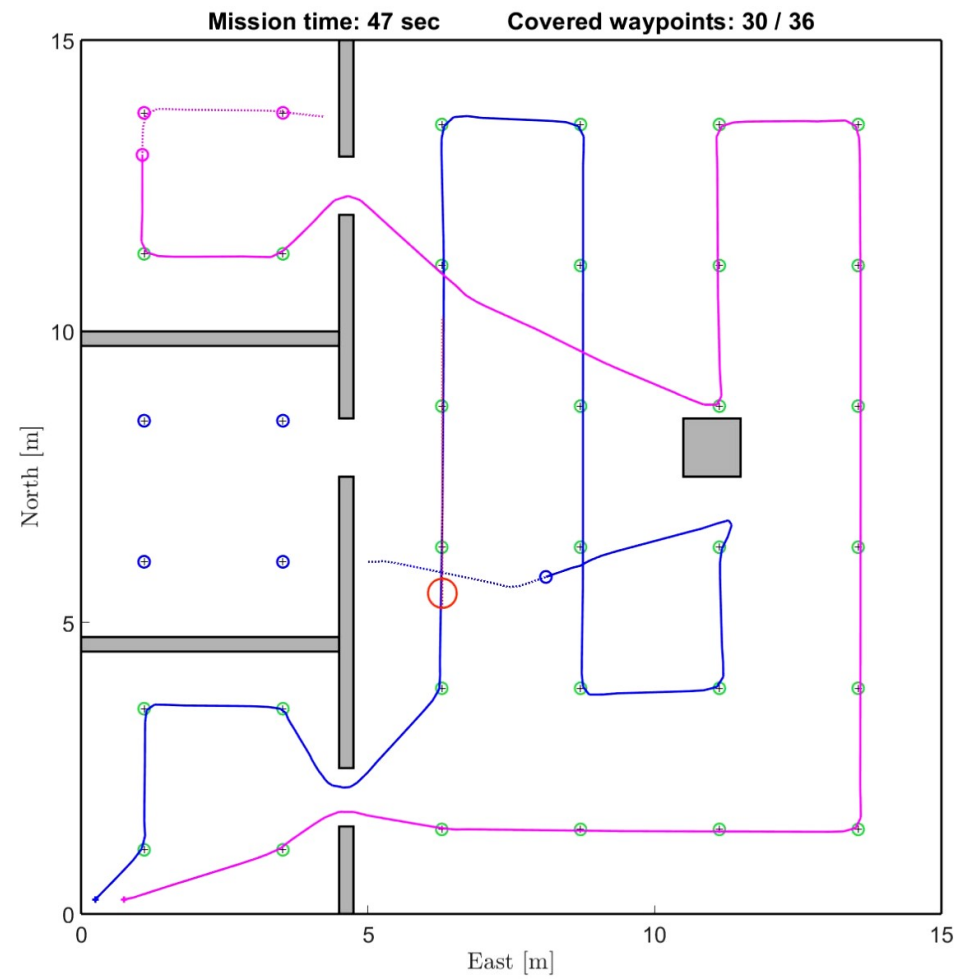
# Goal



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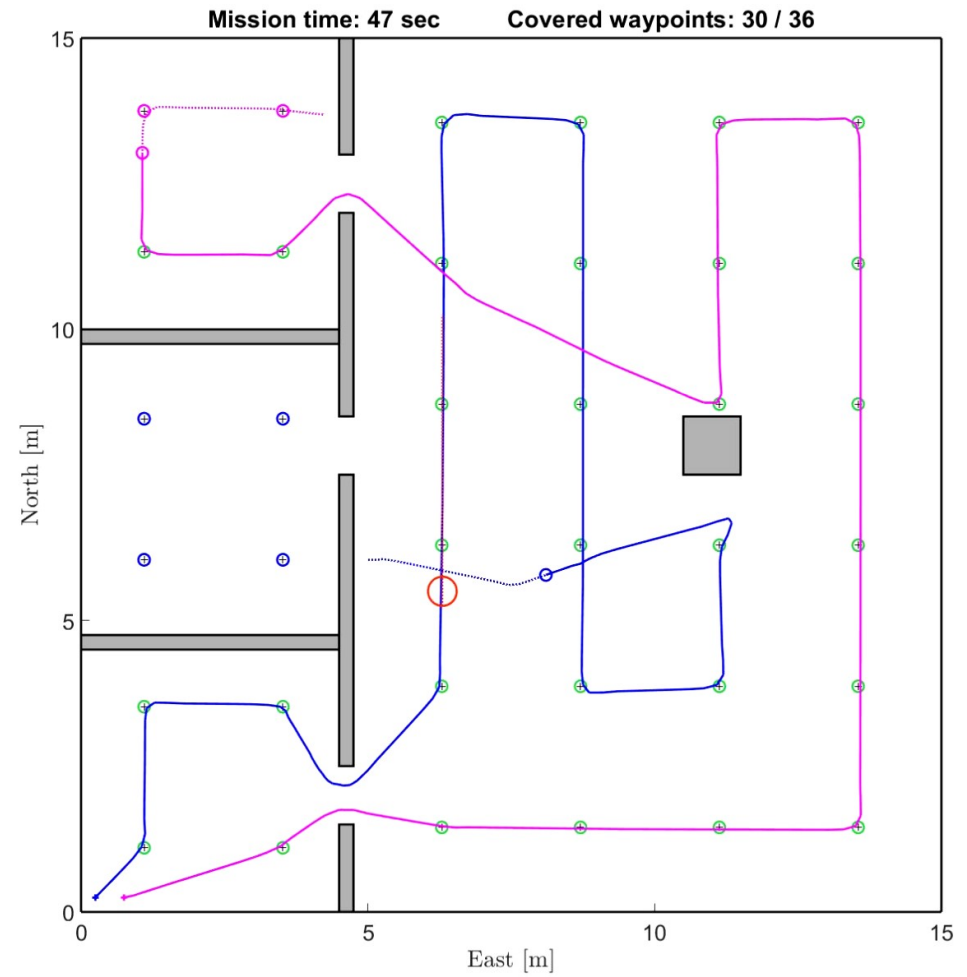
- Cover a specific area using multiple UAS



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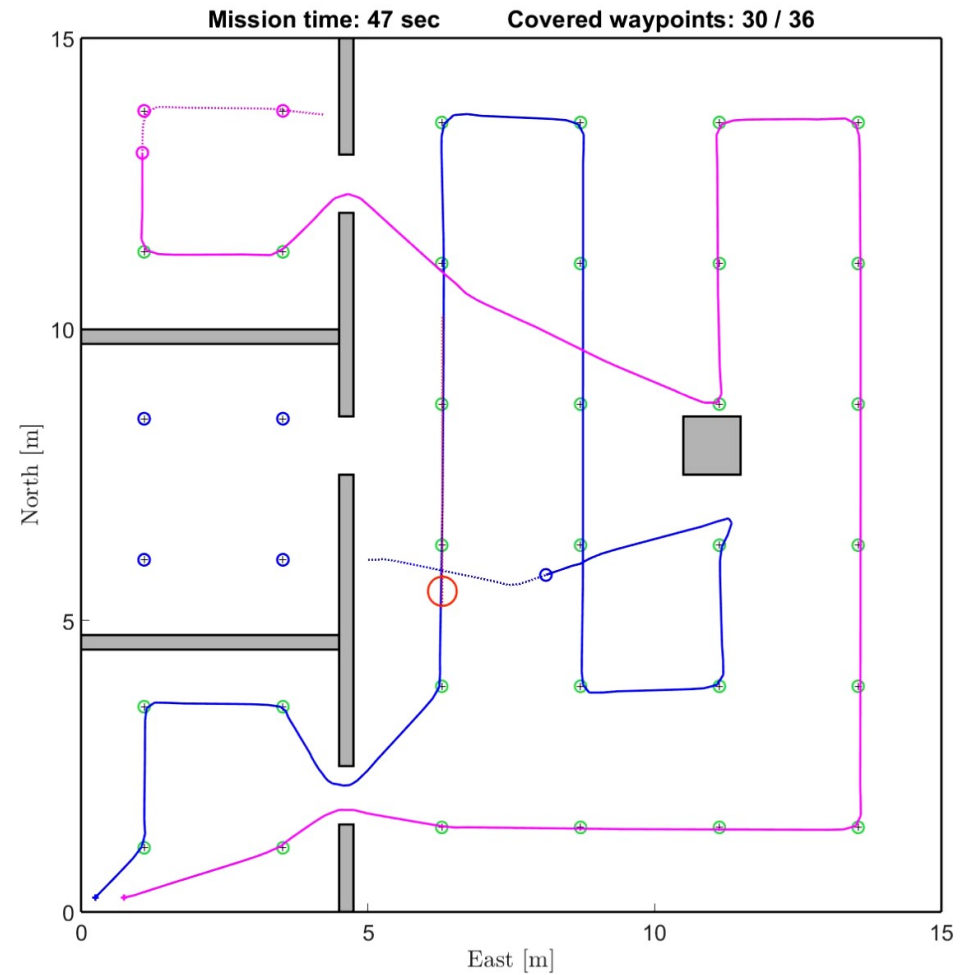
- Cover a specific area using multiple UAS
- Coverage using waypoints



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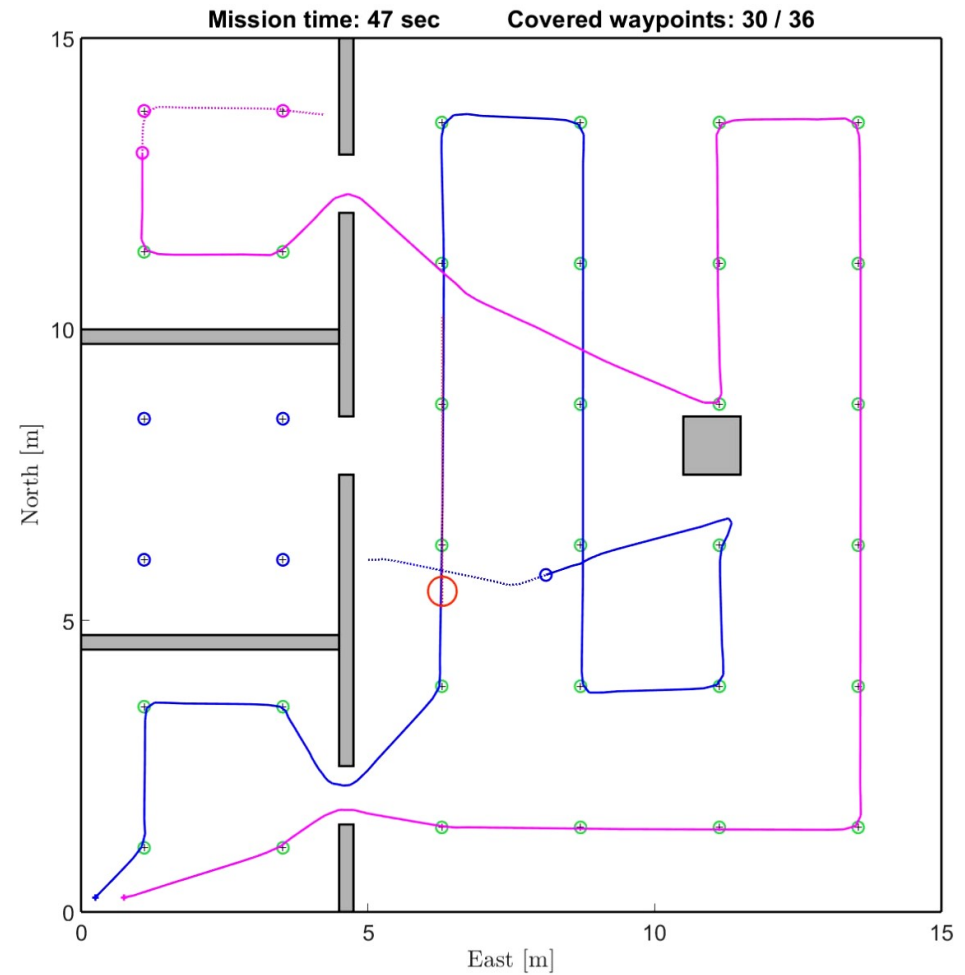
- Cover a specific area using multiple UAS
- Coverage using waypoints
- Fixed obstacles (geofences, buildings)



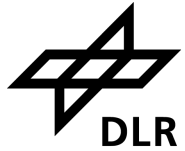
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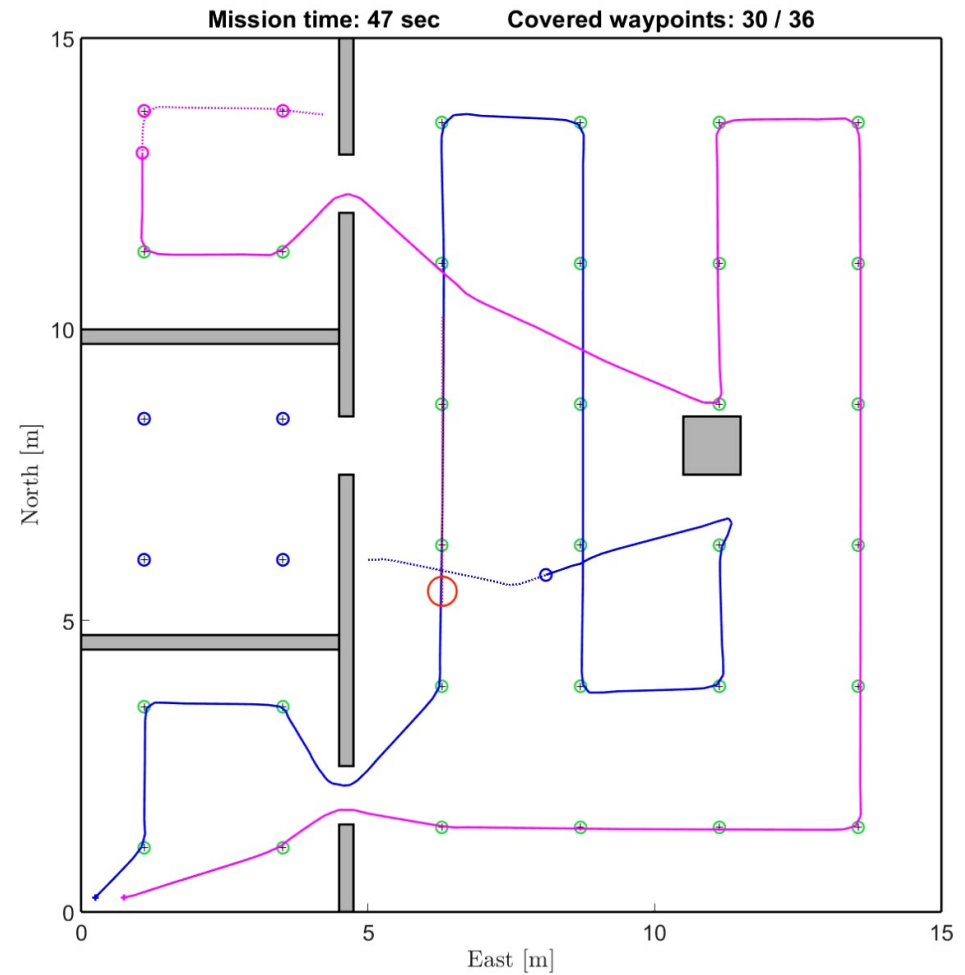
- Cover a specific area using multiple UAS
- Coverage using waypoints
- Fixed obstacles (geofences, buildings)
- Moving obstacles (uncooperative UAS)



# Goal



- Cover a specific area using multiple UAS
- Coverage using waypoints
- Fixed obstacles (geofences, buildings)
- Moving obstacles (uncooperative UAS)
- Cooperative path planning
- Dynamic replanning

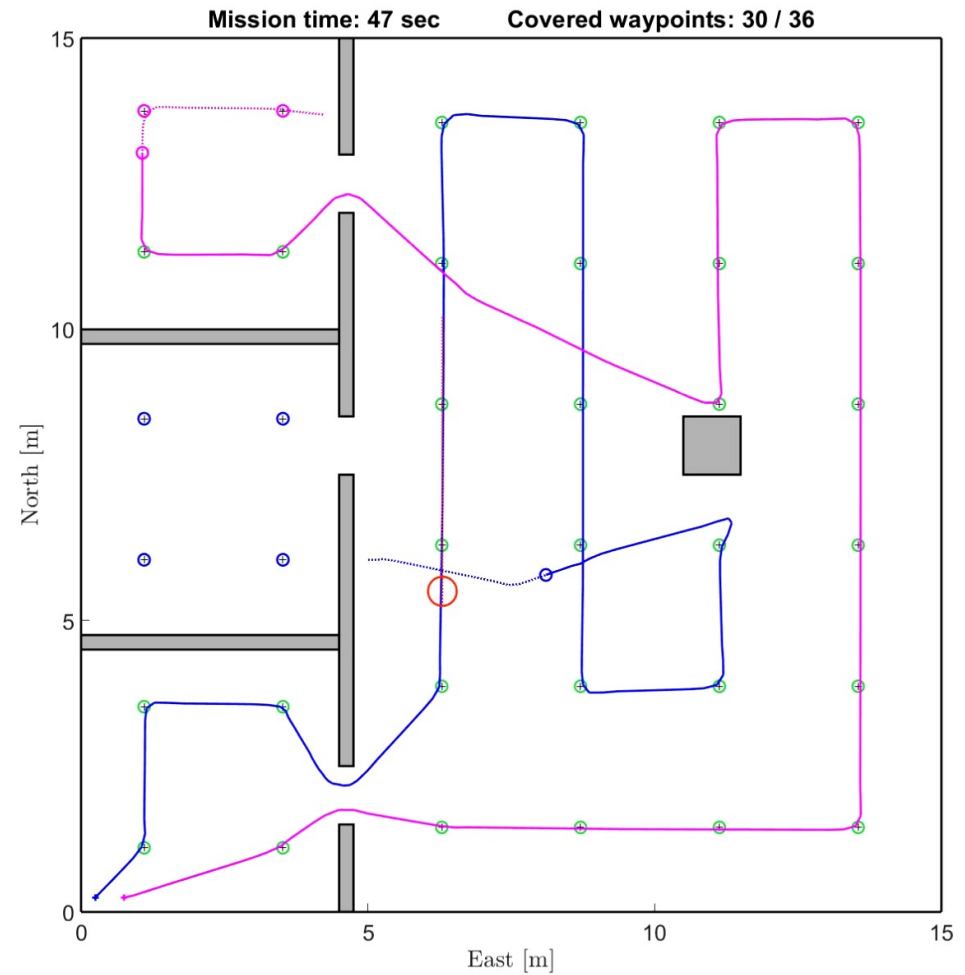




# Goal



- Cover a specific area using multiple UAS
- Coverage using waypoints
- Fixed obstacles (geofences, buildings)
- Moving obstacles (uncooperative UAS)
- Cooperative path planning
- Dynamic replanning
- Considering vehicle dynamics
  - ODE`s
  - Performance model ?

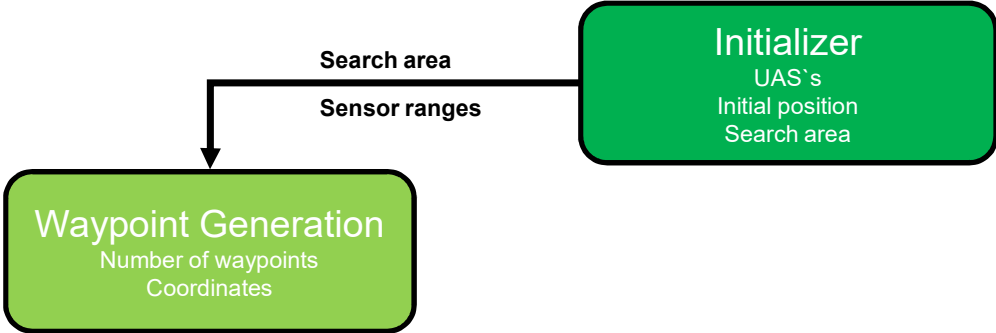




# Scheme



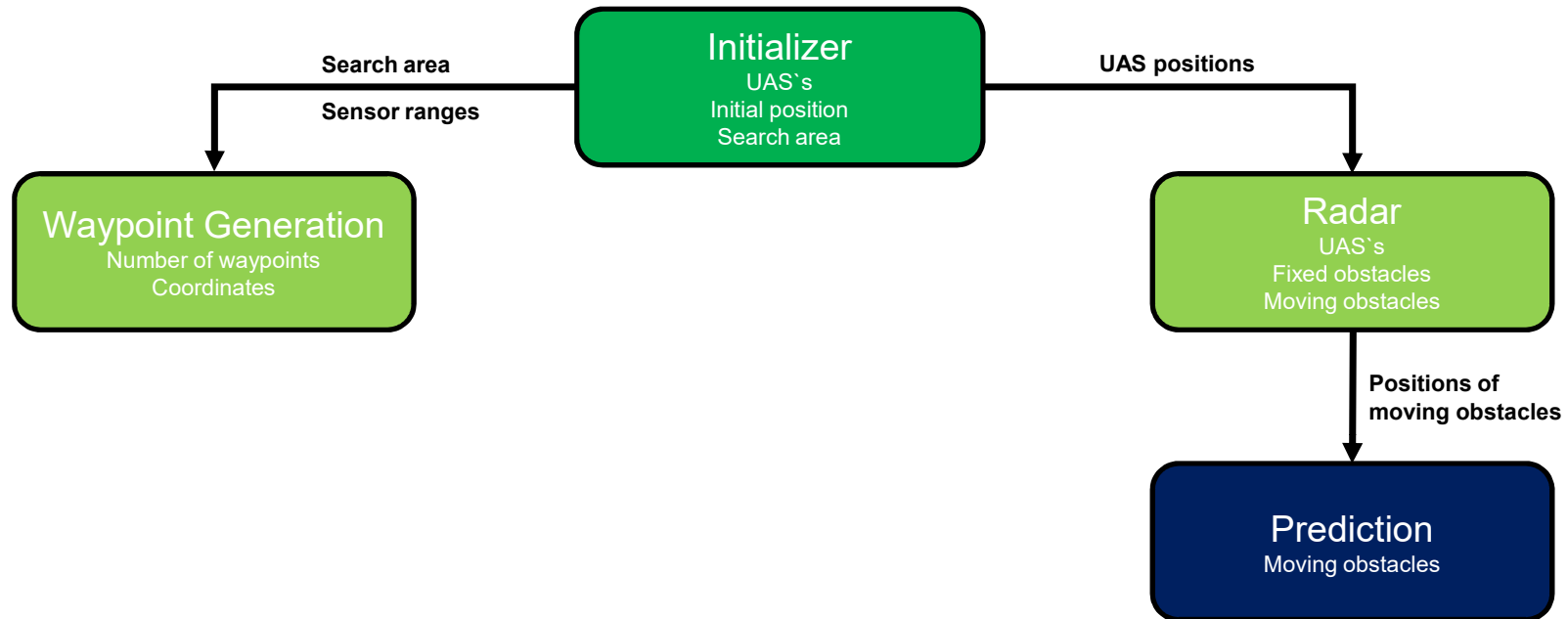
# Scheme



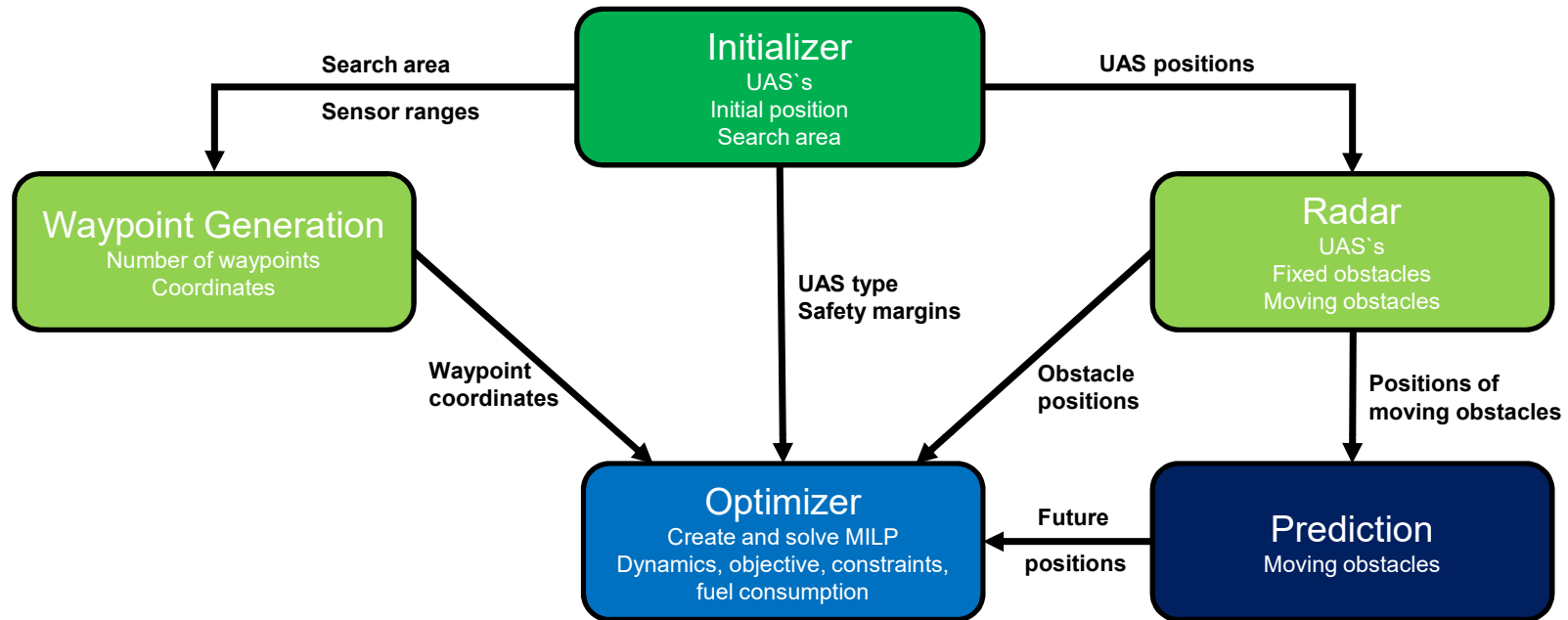
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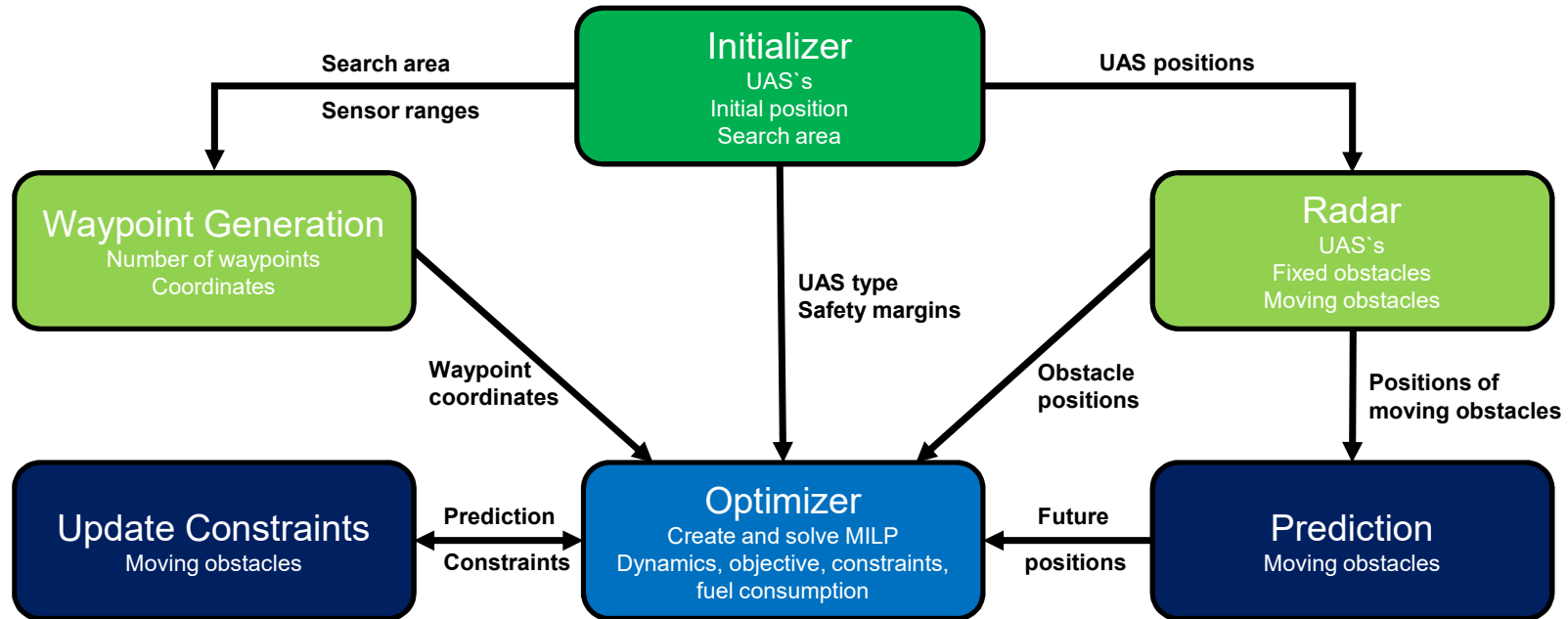
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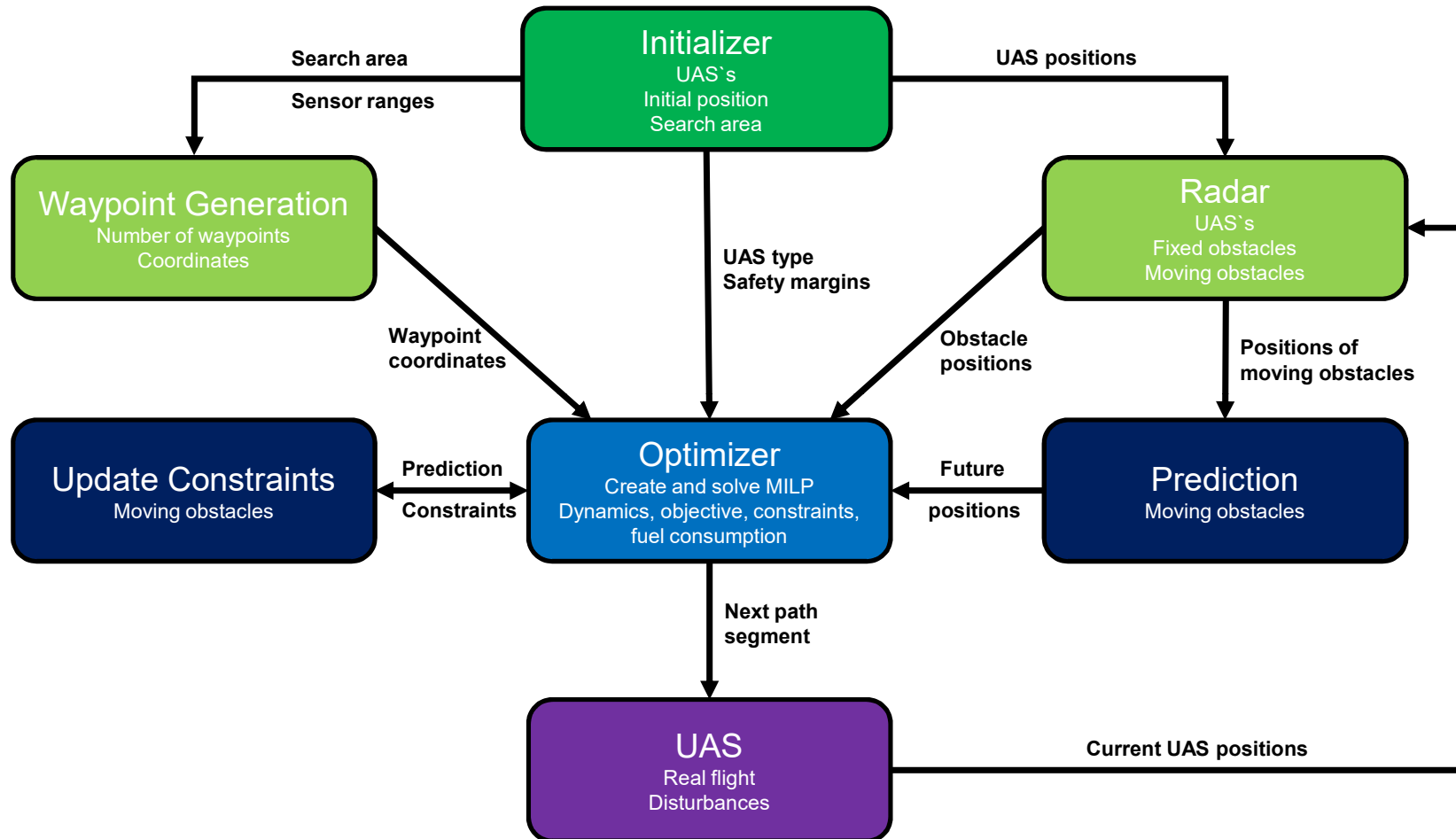
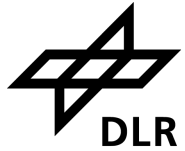
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# Mathematical Setup



# Mathematical Setup



$$\min_{u_i(\cdot), v_j(\cdot)} \sum_{k=0}^{N-1} \left( \sum_{i=1}^{n_{\text{UAV}}} W_{u,i} \cdot |u_i(k)| + \sum_{j=1}^{n_{\text{WP}}} W_{z,j} \cdot z_j(k) \right)$$

$$\text{s.t. } \forall k \in \{0, \dots, N-1\}, i \in \{1, \dots, n_{\text{UAV}}\}, \\ j \in \{1, \dots, n_{\text{WP}}\}$$

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k)$$

$$x_i(k) \in \mathbb{X}_i(k), u_i(k) \in \mathbb{U}_i(k)$$

$$z_j(k+1) = z_j(k) - v_j(k)$$

$$v_j(k) \leq d_j(k)$$

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# Mathematical Setup



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# Mathematical Setup



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- Minimize the cost of energy consumption and penalty terms from uncovered waypoints
- Prediction horizon  $N$ , number of waypoints  $n_{\text{WP}}$ , number of UAS  $n_{\text{UAS}}$

# Mathematical Setup



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- Linearized system dynamics

# Mathematical Setup



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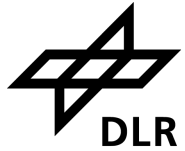
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# Mathematical Setup



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- Linear constraints for the state and input variables
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# Mathematical Setup



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- Prediction horizon  $N$ , number of waypoints  $n_{\text{WP}}$ , number of UAS  $n_{\text{UAS}}$
- Linearized system dynamics
- Linear constraints for the state and input variables
- Waypoint dynamics (0 ~ covered, 1 ~ uncovered)
- Artificial input and constraints

# Mathematical Setup



$$\min_{u_i(\cdot), v_j(\cdot)} \sum_{k=0}^{N-1} \left( \sum_{i=1}^{n_{\text{UAV}}} W_{u,i} \cdot |u_i(k)| + \sum_{j=1}^{n_{\text{WP}}} W_{z,j} \cdot z_j(k) \right)$$

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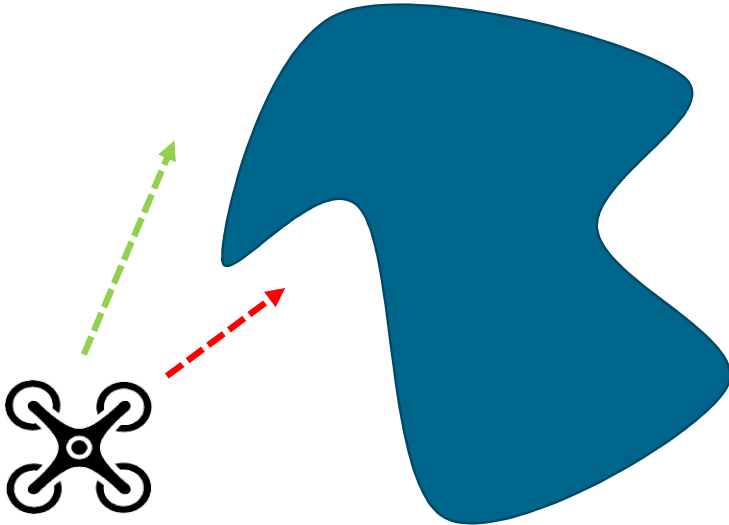
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- Minimize the cost of energy consumption and penalty terms from uncovered waypoints
- Prediction horizon  $N$ , number of waypoints  $n_{\text{WP}}$ , number of UAS  $n_{\text{UAS}}$
- Linearized system dynamics
- Linear constraints for the state and input variables
- Waypoint dynamics (0 ~ covered, 1 ~ uncovered)
- Artificial input and constraints
- Distance condition for the waypoints

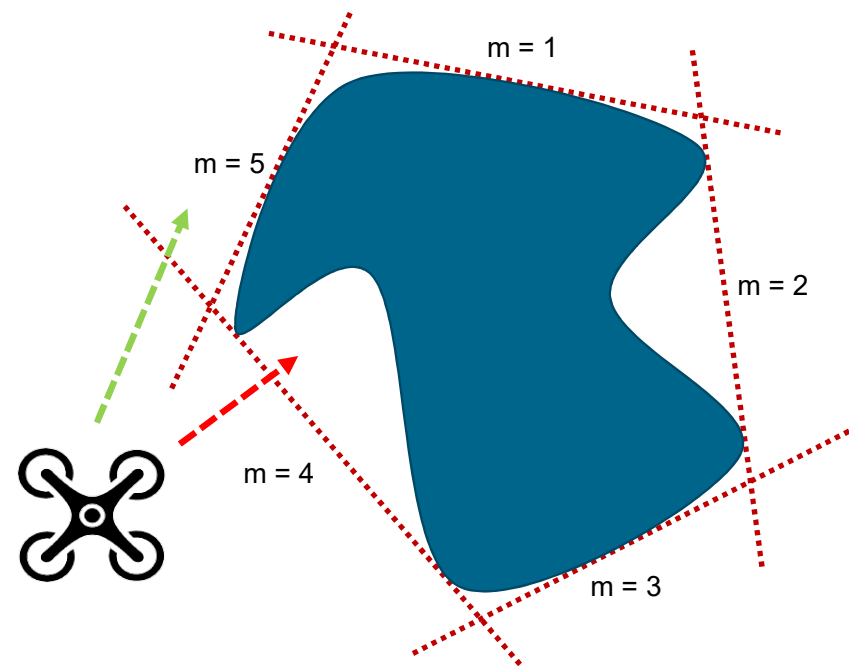
# Constraints



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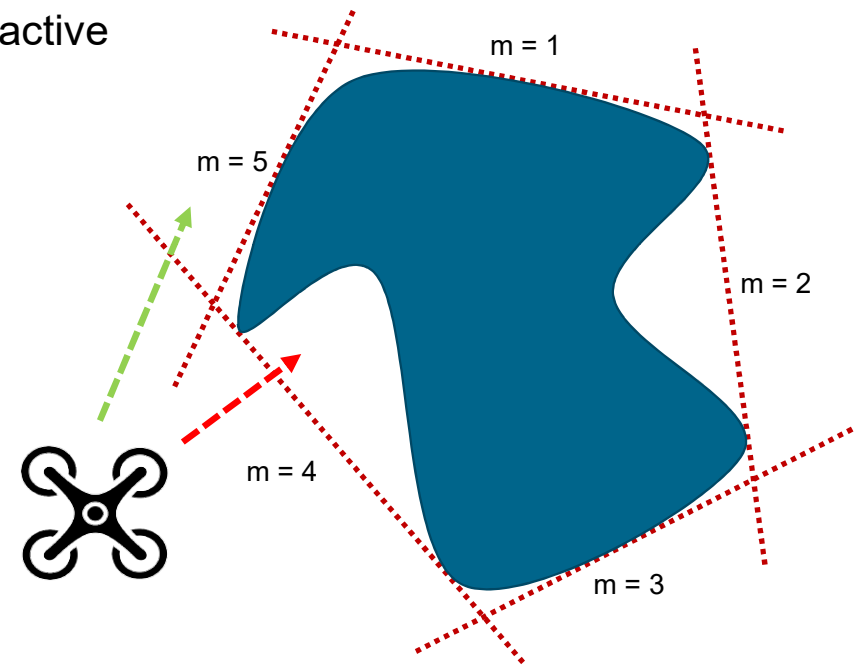
- Nonlinear constraints are approximated via affine functions



# Constraints

- Nonlinear constraints are approximated via affine functions
- Binary decision variables are used to set constraints active/inactive

$$\begin{aligned} & \left( x_{\text{UAS},i}(k) - x_{\text{obs},j}(k) \right) \cdot \cos \left( \frac{2\pi m}{M} \right) \\ & + \left( y_{\text{UAS},i}(k) - y_{\text{obs},j}(k) \right) \cdot \sin \left( \frac{2\pi m}{M} \right) \\ & \geq \delta_{\text{UAS,obs}} - M_{\text{big}} \cdot \beta_{i,j,m}(k) \end{aligned}$$

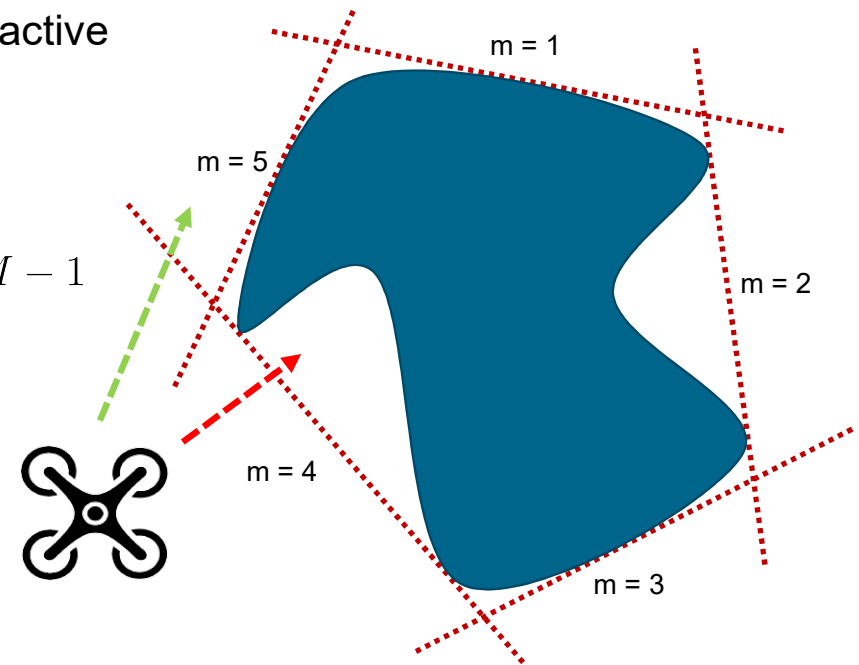


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$$\sum_{m=1}^M \beta_{i,j,m}(k) \leq M - 1$$

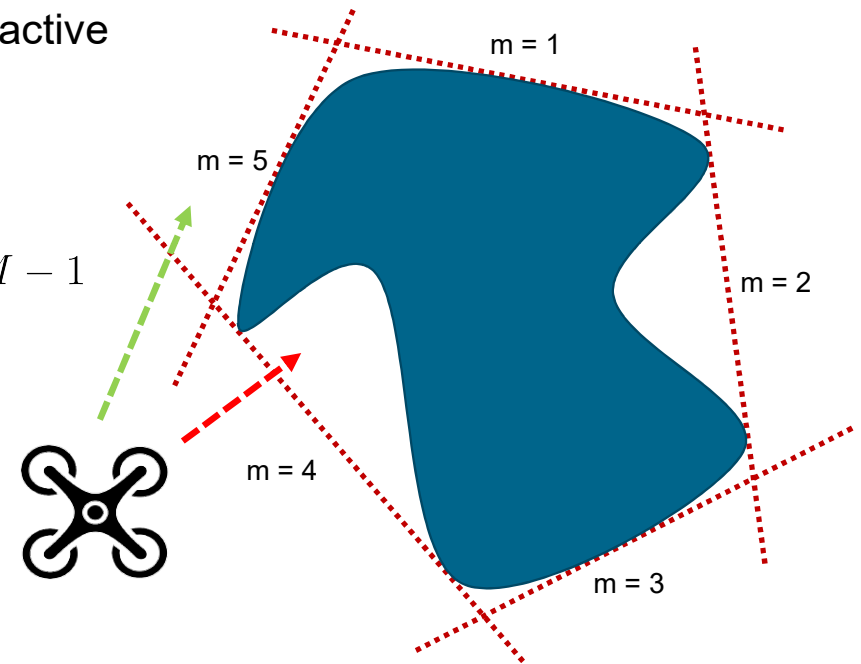


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- Waypoint coverage constraints are handled similarly



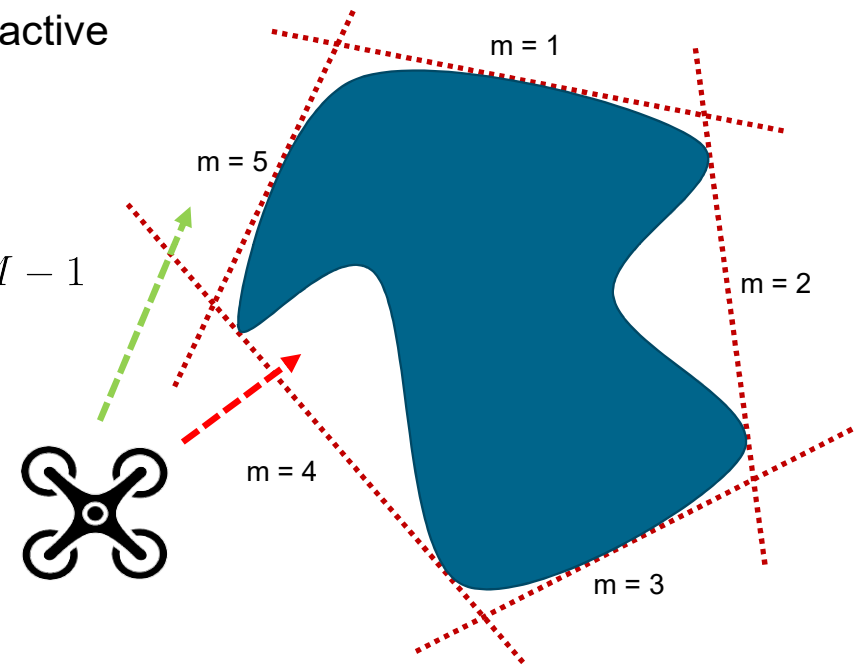


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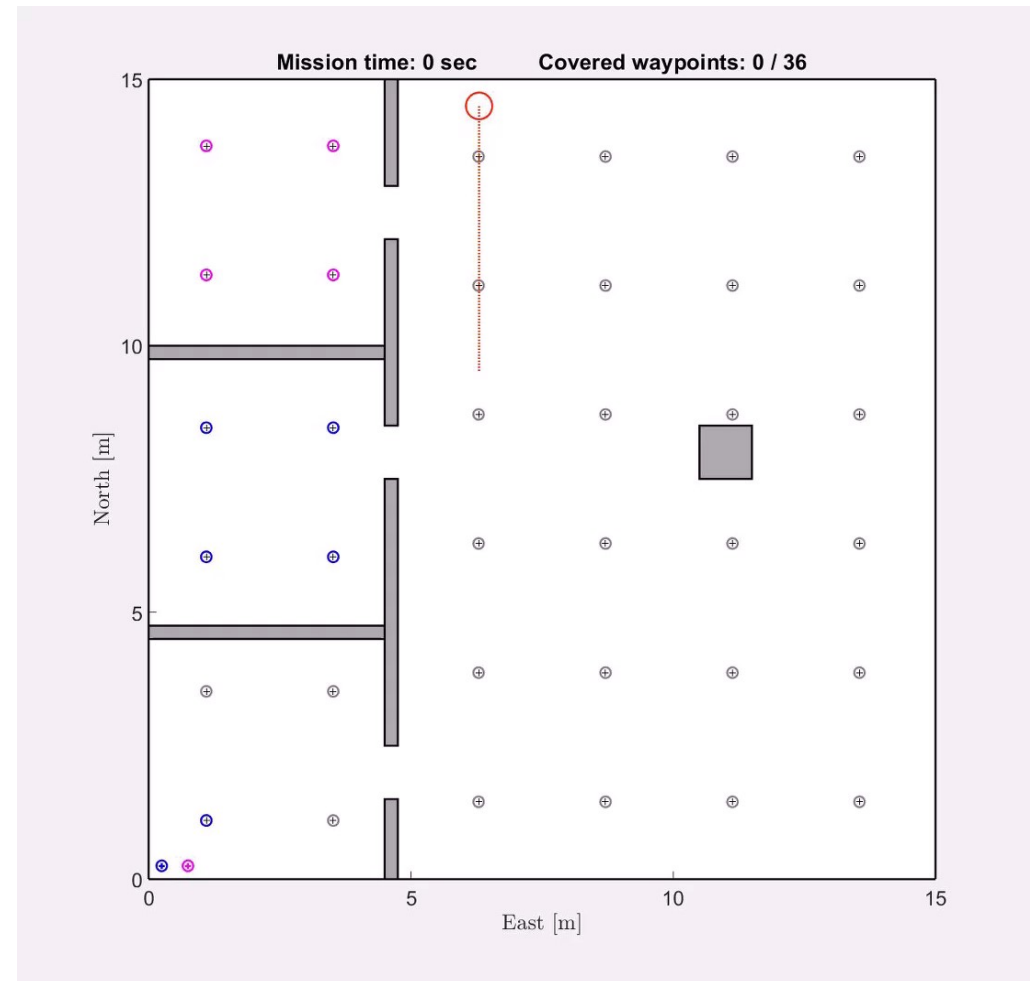
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- Waypoint coverage constraints are handled similarly
- The number of binary variables can be decreased



# Simulation Results

- Area ~ 15 m x 15 m
- Prediction horizon ~ 5 s, 20 evaluation points
- 2 searching quadcopter (blue and magenta), 11 states, 4 input variables
- 1 moving obstacle (red)
- Predictions (dotted)
- 7 fixed obstacles (grey)
- 36 equidistant waypoints
  - Uncovered (grey)
  - Covered (green)
  - Only one drone can cover (blue, magenta)



# Simulation Results

